MATHEMATICS MATTERS – A CRUCIAL CONTRIBUTION TO THE COUNTRY'S ECONOMY

National Science and Engineering Week Seminar on Thursday 15th March

THE MATHEMATICAL SCIENCES LANDSCAPE IN THE UK

Professor Sir Adrian Smith
Director General, Knowledge and Innovation, Department of Business Innovation and Skills

As a mathematician in Government, I want to use my contribution to the P&SC *Mathematics Matters* seminar to highlight the Government's commitment to the subject (which is inseparable from its wider support for science and research). I also wanted to fly the flag for UK mathematical excellence – at the same time I sounded some warnings about the challenge to maintain this in the context of international developments and competition.

The evidence is there that Government as a whole gets the importance of mathematics and science: look at the Spending Review settlement and successive announcements of further capital funding since (Budgets, Autumn Statements etc). Neither is moral support for the growth of the subject lacking. In a recent speech at a British Academy conference on quantitative skills David Willetts stressed not only the increasing importance of statistical literacy for those studying non-STEM disciplines but also the value of mathematics to every one of us in our daily lives.

There are other reasons to be cheerful. We have seen a steady rise in the take-up of mathematics over the last 10 years at A Level – linked to which has been a corresponding growth in B plus grades. Entrants to final degrees show a similar trajectory.

These figures evidence the benefits of a sustained Government commitment to the subject but they also reflect another fact, borne out by the other speakers at this event and my own experience in teaching the subject for thirty years: in the fields of mathematics and science… *Britain Has Talent*.

Note our national research standing. With 3.9% of World Researchers and 3.0% of World Gross Expenditure on R&D (GERD) the UK delivers 6.4% of articles. These articles have 9.4% of article usage, gain 10.9% of citations and comprise 14.0% of the top 1% highly-cited articles. In terms international citation of UK articles on mathematics we vie with the US as a world leader and consistently surpass the lands of Descartes and Gauss.

> . . . Mathematics is vital to the formation and operation of policy across Government, from health and education to defence and national security. . .

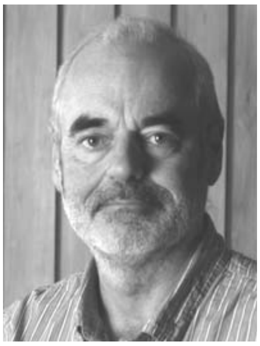
But the same figures are both 'sweet and sour', revealing as they do in a context of increasing international competition that China is coming up fast.

When I was doing my PhD in statistics it was most unusual to see a Chinese name on a research paper; now it is most unusual to see a statistics journal without Chinese names, sometimes in the majority.

Yes, we ***can*** compete but I also believe that International co-operation is one way of responding to competition, forming partnerships with a global talent pool. It is happening already: 46% of UK papers in 2010 had an international co-author; higher than any other G8 or BRIC country except France. Papers with an international co-author achieve twice as many citations as those produced within a single institution, showing that collaboration drives up both quality and impact.

Mathematics is vital to the formation and operation of policy across Government, from health and education to defence and national security. While the climate of increasing competition and tight public funding means we must prioritise within our research portfolio, the importance of mathematics is assured.

ASSESSING AND COMMUNICATING RISKS AND UNCERTAINTY



Professor David Spiegelhalter
Winton Professor of the Public Understanding of Risk, University of Cambridge

Every day we get exposed with messages about risk – the media love a good scare story. At our website understandinguncertainty.org we try to take apart some of these stories using maths. For example, a few weeks ago a study from Harvard on the dangers of eating red and processed meat got a lot of attention, with the Daily Express reporting that if we ate less meat *"10% of all death could be avoided"* – a truly remarkable claim revealing they had no idea what the study actually said. This was that a daily portion of red meat was associated with an increase in the *annual* risk of death by 13% over the period of the study. But even if this number were true – which can be disputed – what would this mean for life-long meat-eaters? Would it mean a 13% shorter life?

To check this we need to consult the life-tables produced by the Government Actuary and apply the 13% extra risk for each year of eating meat. This uses the mathematical statistical technique of 'survival analysis', and reveals that a 40 year-old man who eats a quarter-pounder burger for his working lunch each day can expect, on average, to live to 79, while his mate who avoids the burger can expect to live to 80.

Put this way, the lost year associated with the burger-stuffing does not sound too bad if, in the classic words of Kingsley Amis, self-denial means an extra year in a nursing home in Weston-super-Mare. But we can reframe the message in a more alarming way: over a lifetime habit, each daily portion of red meat is associated with about 30 minutes off your life expectancy – more than the time it takes to eat it – and around the same as 2 cigarettes or each day of being 5 Kg overweight. This idea of accelerating your daily ageing may be more persuasive to change behaviours.

Mathematical risk models are very widely used: in insurance and pensions, finance and economics, individualised risk assessments for heart attacks, health policy by NICE and for epidemics, weather and climate and associated hazards of flooding and so on. The National Risk Register produced by the Cabinet Office has become increasingly sophisticated and now publicly communicates the assessed numerical chances (except for security events) of various extreme scenarios over the next 5 years, such as severe space weather and Icelandic volcanic eruptions.

Making such assessments is tricky, and the NERC is now funding the PURE initiative that brings together mathematicians, statisticians and environmental scientists to develop risk models for natural hazards as well as appropriate means to communicate the results. One attractive metaphor for communication involves the idea of "possible futures", which can be based on *Monte Carlo* methods in which large numbers of possible future 'worlds' are simulated under slightly different conditions, and the proportion in which a particular extreme event happens reflects the chances of the event occurring. These techniques started in the US hydrogen bomb project, and the UK is now world-leading in theory, software and applications, including the use of 'ensembles' for weather forecasting. Unfortunately there is still a reluctance to communicate publicly the chances of different weather patterns, although in the US 'possible paths' of hurricanes are routinely shown on public news broadcasts.

The Bank of England is an organisation that has fully embraced the open communication of uncertainty about its forecasts, with its 'Fan Charts' expressing what might be expected *"If economic circumstances identical to today's were to prevail on 100 occasions …Consequently, GDP growth is expected to lie somewhere within the entire fan on 90 out of 100 occasions."* A central prediction line is deliberately not given for a week after the initial release of the fans, much to the annoyance of the press, who are prevented from declaring a single 'prediction' for growth and inflation.

All science is hedged with uncertainty, and when difficult policy decisions have to be made it is a tricky balance to be both upfront about uncertainty and retain trust. Nevertheless, when the Commons Science and Technology Select Committee examined *Scientific Advice and Evidence in Emergencies,* David Willetts – Minister for Universities and Science – said that *"Communicating the intrinsic uncertainties in scientific advice is something that we probably need to do better".*

Mathematical risk models are a vital tool in this process, but the financial crisis has shown that they should be accompanied by a warning: there are serious dangers if fancy mathematical tools are used by people who do not understand their limitations. The solution is both to invest in mathematics, and to make full use of the great expertise we have this in this country.

MATHEMATICS IN SECURITY AND INTELLIGENCE

Malcolm MacCallum
Director, Heilbronn Institute for Mathematical Research, Bristol

As the successor to the famous Bletchley Park, Government Communications Headquarters' (GCHQ) mission relates to national security, the economic wellbeing of the UK and support of the prevention or detection of serious crime, as set out in the Intelligence Services Act 1994.

The Heilbronn Institute is a partnership between GCHQ and Bristol University. It continues part of the Bletchley tradition – exploiting the skills of top-class UK academic mathematicians in GCHQ's service.

Part of GCHQ's role is providing advice and assistance about cryptography and other matters to protect UK information and other material (ie communications and data) from those who might wish to eavesdrop, steal, corrupt or deny access to it. Such security has become urgent with the 'cyber-security' challenge, arising from the global internet. The Foreign Secretary has spoken publicly about this, most recently at the London conference on cyberspace last year.

I can only use historical examples, but there are good modern parallels – indeed surprisingly many.

Secure transmission of financial information to websites (those starting with https://) relies on 'public key cryptography', first discovered at GCHQ. Its conceptual framework was due to James Ellis and the first implementation to Clifford Cocks, a new recruit from a postgraduate mathematics degree course, who invented the key idea in about 30 minutes. This was possible because of his good knowledge of relevant mathematics: the security depends on the quantifiable difficulty of factoring a product of two very large prime numbers. The transmission of your credit card details to secure web sites is made safe by mathematics (though this protection does not extend to other types of fraud).

The second example comes from Bletchley. Colossus, the first computer, was built to decode teleprinter messages between German Army HQs encrypted by Lorenz machines, codenamed Tunny. Tunny added a "key" to the unencoded input. From a 'depth' (two messages with related content and the same key) Colonel John Tiltman extracted 3976 characters of key. He gave it in autumn 1941 to Bill Tutte to analyse. Tutte was a Cambridge postgraduate chemist with an interest in mathematics and trained in cryptography. From the key, Tutte inferred the structure of the Tunny machine, although, unlike the famous Enigma, it had never been captured or seen.

This information was enough to enable much codebreaking. The mathematician Max Newman saw that mechanised methods were needed to do better. Tutte's second major invention was a statistical method of finding Tunny wheel settings directly from the coded message, and Newman had Colossus built to do this: Colossus was a special purpose cryptographic device, rather than a general purpose computer. The design, by Tommy Flowers of the Post Office, incorporated several novel features.

Tunny decryption was very important, although it gave many fewer messages than Enigma. It allowed us to forewarn the Russians of the German attack around Kursk in July 1943, decrypt messages direct from Hitler himself in 1944, know the German dispositions before D-Day, and assess the value of the Italian campaign in tying down German forces. The work of Tutte and Flowers was arguably an even greater achievement than that on Enigma.

Such work, then and now, depends on a vibrant research culture in mathematics in UK universities.

GCHQ applies the skills people bring: Turing, Newman, Tutte, Flowers and Cocks provide examples.

UK mathematics, especially pure mathematics, makes a major contribution to UK security and intelligence.

My colleagues and I intend to ensure this is considered in the forthcoming Research Excellence Framework.

It is the experience of solving mathematical research problems, being able to adapt that experience to new questions, and work in teams, which we need, and we exploit the variety and geographic spread of UK expertise. We therefore want to ensure that the difficulties highlighted in the article by Professors Glendinning and Brown (Science in Parliament 68 (4), 30) are overcome.

GCHQ and HIMR each put resources into supporting the desired UK research culture. For example, we part-sponsor various undergraduate and postgraduate events; employ undergraduate and postgraduate interns; fund targeted postgraduate studentships and postdoctoral fellowships; and sponsor academic workshops and conferences.

We want to enable talented UK students to progress into academia. Concerns include the impact of future funding structures, making UK postgraduate training rigorous enough to compete with the EU and US, and ensuring sufficient postdoctoral positions (which provide the vital bridge between postgraduate study and an academic career).

Ending by putting on my hat as President of the International Society on General Relativity and Gravitation, I was recently fascinated to see, on a Kent cereal farm, expensive equipment whose accurate use depended on the corrections from general relativity to the GPS (Satnav) system. Einstein in 1915 could not have foreseen this impact of mathematical physics, another area with endangered funding, on farming.

COUNTING CASES:
How does mathematics help us control infectious diseases?

Déirdre Hollingsworth
Junior Research Fellow, MRC Centre for Outbreak Analysis and Modelling, Department of Infectious Disease Epidemiology, Imperial College London

Infectious disease transmission is a dynamic process, resulting from a sequence of chance events. Infectious people make contact with people who are susceptible to the disease, whether through sharing a bus, making conversation or through more intimate contacts, and transmission occurs with a certain probability. The outbreak grows as more people become infected until eventually the number of people who haven't been infected becomes small and the epidemic runs out of steam. But how do we know how many cases to expect? How many people do we need to vaccinate to prevent an outbreak? If we can't prevent an outbreak, how should we use our resources to prevent the hospitals becoming overwhelmed? These crucial public health questions can only be addressed using mathematical methods and analyses.

Mathematical models are a way of rigorising our thoughts about infectious disease transmission, expressing the transmission process in a formal language. The development and analysis of models is ideally an interaction between clinicians, biologists, policy makers, statisticians and mathematicians. We can inform these models with the available data, using modern statistical methods, and extrapolate our insights to designing 'what-if' scenarios for policy.

Simple, yet powerful, insights can emerge from these mathematical constructs. For example, many core results in infectious disease epidemiology come from the concept of the basic reproductive number (R0), which is the mean number of new infections caused by a single infected individual in a wholly susceptible population (see figure). If individuals at the start of an epidemic infect on average more than one person ($R0>1$), then the epidemic will grow.

How does this help with designing control measures? In the case of vaccination, vaccinated individuals cannot be infected, and so the effective reproductive number in the presence of vaccination is lower than without vaccination (see figure). In fact, if p is the proportion of the population who are vaccinated, then on average the number of new infections which can be caused by a single infected individual is $(1-p)R0$. If the aim of the policy is to prevent an outbreak by vaccinating the population then the average new infections needs to be less than one, $(1-p)R0<1$. We can rearrange this expression to get the critical vaccination proportion $pc>1-1/R0$. If the average number of new infections in an unvaccinated population is large, then the critical vaccination proportion is high (eg measles, which has an $R0>10$ and so more than 90% of the population need to be vaccinated to prevent transmission), whereas the critical fraction is smaller for a disease like smallpox with a $R0 \sim 4$, which facilitated the eradication of smallpox by vaccination. The most important insight in this analysis (most famously outlined by Karl Dietz in 1975) is that the whole population does not have to be vaccinated to control an outbreak of an infectious disease – ie the critical vaccination proportion is not 100%. This is because those who are unvaccinated are protected by the vaccination status of their contacts, which prevent infections which would otherwise be amongst their contacts (so called 'herd immunity').

Vaccination strategies are, of course, based on a more nuanced understanding of disease transmission and vaccine uptake than this scenario suggests. For example, an important consideration for a childhood vaccination programme is the likely impact on transmission between children and adults. More complex models, together with high quality data, are used to inform the details of policy in particular diseases.

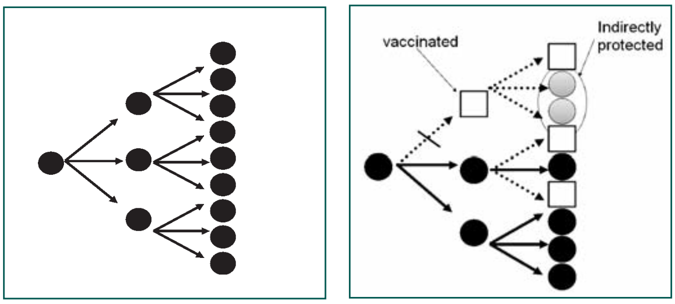


Figure legend
Infectious disease transmission occurs in a chain, with each infected individual infecting a number of others (in this schematic exactly 3 others, left figure, black circles). If vaccination is introduced (right figure, white squares) the average number of new infections drops (here the average is 2 when $4/13 \sim\sim 1/3$ of the population is vaccinated). The vaccinated people are not the only ones who are protected, there is indirect protection for a proportion of the population. This means that not everyone needs to be vaccinated to eradicate a disease (see text for more details).

There are many other forms of complexity which can be introduced. For diseases like malaria, which are spread by mosquitoes, our models take account of rainfall and ecology in local areas to estimate the impact of the use of bed-nets on transmission. For sexually transmitted diseases, explicit modelling of sexual contact networks may be required. Since the problems addressed within this field are so varied, the methodology we use comes from a number of mathematical research areas including probability, statistics, mechanics, ecology, network theory, enzyme kinetics and computation, from both long-standing results and current areas of active research.

All models have their limitations and the mathematicians who use them are all too aware of these frailties. The impact of uncertainties within the model and the data depends on the question being asked. Models can therefore rarely be used 'off-the-shelf' when addressing a new policy question without some understanding of these limitations and so expert users are required.

The UK is a world leader in the field of infectious disease modelling and in the interaction between modelling and public health policy. In recognition of this, the Medical Research Council Centre for Outbreak Analysis and Modelling, led by Professor Neil Ferguson OBE, has recently been designated the first World Health Organisation Collaborating Centre for infectious disease modelling. The success of the UK in this field is based on a history of using the most appropriate mathematical or statistical tool for the problem at hand, and as such is reliant on a rich supply of novel mathematical research.

MATHEMATICS MATTERS

MATHEMATICS OF INFORMATION

Professor Jared Tanner
Professor of Mathematics of Information, University of Edinburgh

The proliferation of computing and data processing has influenced the past two decades so fundamentally as to inspire the term 'Information Age'. Although collecting and analysing data has served for centuries as the bedrock of the scientific method, it is the introduction of computers that has elevated the scale of the data to that which we commonly process today. Mathematics, and in particular statistics, has always played a central role in the collection and analysis of data, with foundations laid by leading UK researchers such as Bayes, Pearson, and Fisher. The prevalence of large data sets and the great need for algorithms which can extract meaningful information has resulted in a resurgence of seemingly pure mathematics research engaging with statistical questions. Two such exemplary novel topics are compressed sensing and matrix completion.

The ease with which we digitally store multitudes of documents, as well as capture and transmit images and video, relies on the expectation that the data we are interested in can, in some way, be compressed to a much smaller representation. For instance, most documents are composed of relatively few different words, and most images are composed of few objects, each of which can normally be approximated by slowly varying colours. Nonetheless, we usually treat compression as a secondary process, to be performed only after painstakingly acquiring very detailed information, such as high-precision images. It is, obviously, highly inefficient to first carefully acquire information when it is known beforehand that only a compressed version will be retained as it contains the essential information. This suggests the question: can we acquire the desired essential information more efficiently by including compression into the measurement process? If so, such a method should allow for dramatically more efficient sensors of all sorts.

Remarkably, seemingly pure mathematics informs us that this is indeed possible, generating a paradigm shift in data acquisition. For instance, it is common in medical imaging that there is limited ability to acquire information; limitations range from the inability to hold one's breath longer than a short time during an MRI scan to limiting radiation exposure. Incorporating compression into the data acquisition process – known as compressed sensing – has resulted in seven times faster MRI scan rates. Mathematics has both inspired this approach and has proven its effectiveness.

Similar notions are also being applied to questions in the digital economy. The extraordinary quantity of information available has made search and recommendation software essential. For example, the success of online video rental companies is driven not just by the quantity of films available by a company, but also their ability to make recommendations to their customers. Mathematically this task corresponds to having a large spreadsheet of different customers and their film choices; however, the spreadsheet is largely empty as most customers have seen a tiny fraction of existing films. The company is tasked with predicting how customers would rank unseen films, and then using this information to make recommendations. This process is referred to as "matrix completion", and is closely connected to compressed sensing. Again, the success of this approach relies on the information having an underlying structure that is simple, which might correspond to there being relatively few types of film watching preferences.

The underpinning mathematical theory of compressed sensing and matrix completion follows directly from the following, apparently abstract, question. Consider a triangle with sides of equal length, a cube with all sides equal, or the object one gets when adjoining the bases of two pyramids; these are examples of two and three dimensional objects known as Platonic solids. Mathematics allows us to define similar objects in an arbitrarily large dimension. However, only these three objects retain their structure in arbitrary dimensions, making them fundamental to the study of geometry. These objects have undergone intense investigation, including asking questions about which of their properties remain when these objects are 'randomly flattened'. These retained properties in this last question make possible the seven-fold faster MRI scan rates mentioned previously. Much of the foundational theory was developed in the UK, by geometers including: P McMullen, H Ruben, and G Shephard. These, and other, researchers developed the theory of randomly projected objects, and the formulae necessary to calculate when the needed properties would be retained, which now allow engineers to design the next generation of imaging protocols. Even more abstract, nonlinear, geometric questions underlying matrix completion are currently under intense investigation by UK mathematicians. Application inspired interactions bridging mathematics, informatics and statistics portend a wealth of new technological advances.

Presentation to Peter Simpson
Immediately after the Seminar Andrew Miller MP made a presentation to Peter Simpson who stood down as Scientific Secretary and Editor of Science in Parliament on 31st March. Andrew expressed the Committee's gratitude to Peter for all his hard work over the years.
Courtesy of Jonathan Tickner and the Council for the Mathematical Sciences

RECOGNISING THE ROLE OF TECHNICIANS

Jon Poole, Chief Executive IFST

The day-to-day running of the UK, as elsewhere in the world, is increasingly reliant on technology. The changing economic landscape, and increasingly global marketplace, has added even sharper focus to the critical role technical skills play in supporting all business sectors ... and it is no longer only an issue confined to engineering, manufacturing and science industries. Surprisingly, the largest growth in demand for technological skills is now seen in media & publishing, public administration, service and defence sectors. [1]

Although demand for technical skills in the UK is rapidly increasing, recruitment of people into technical roles is failing to keep pace. Today it is calculated that some 2 million people are employed in technician-based roles across all sectors of the economy within the UK.

For the UK to keep pace with demand and hold its competitive position, it is essential to recruit, train and retain technicians in greater numbers than in the past. We need around three pupils out of every senior school class opting for a career in technology. Not only that, there is also a need to encourage more women into technician-based roles.

Against this backdrop, in 2010, Lord Sainsbury brought together a group of interested parties to consider outputs from two White Papers [2] which considered the future needs for scientific and engineering skills. Following on from this, a new body – the Technician Council – was formed to address the underlying issues behind the skills shortage and to look into how a common framework for professional recognition could be provided across science, engineering, IT and health sectors.

This body, Chaired by Stephen Holliday, CEO of National Grid, was constituted of representatives from a wide range of stakeholders including the Science Council; Engineering Council; EngineeringUK; the National Apprenticeship Service; representatives from a number of individual professional bodies as well as key SET employers including Ministry of Defence; Microsoft; Lonza Biologics and BAE Systems.

The challenges facing the Technician Council and its constituent members were complex. Quantifying the numbers employed in technician-based roles was far from straightforward. Science and engineering companies employ many non-technical people and conversely, many technicians work in non-engineering or science based sectors such as food and retailing. Job titles themselves provide no help with the term 'technician' used indiscriminately – ironically often to add status to relatively non-technical roles.

A further challenge facing the Technician Council was the gender imbalance. Women make up 49% of the economically active workforce in the UK, however they remain significantly under-represented at every level in SET employment (Science, Engineering & Technology) – and in higher levels of STEM